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DECISION MAKING UNDER SEVERE UNCERTAINTIES FOR FLOOD RISK MANAGEMENT: A CASE STUDY OF INFO-GAP ROBUSTNESS ANALYSIS

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Flood risk analysis is subject to uncertainties, often severe, which have the potential to undermine engineering decisions. This is particularly true in strategic planning, which requires appraisal over long periods of time. Traditional economic appraisal techniques largely ignore this uncertainty, preferring to use a precise measure of performance, which affords the possibility of unambiguously ranking options in order of preference. In this paper we describe an experimental application of *information-gap theory*, or info-gap for short to a flood risk management decision. Info-gap is a quantified non-probabilistic theory of robustness. It provides a means of examining the sensitivity of a decision to uncertainty. Rather than simply presenting a range of possible values of performance, info-gap explores how this range grows as uncertainty increases. This allows considerably greater opportunity for insight into the behaviour of our model of option performance. The information generated may be of use in improving the model, refining the options, or justifying the selection of one option over the others in the absence of an unambiguous rank order. Secondly, we demonstrate the possibility of exploring the value of waiting until improved knowledge becomes available by constructing options that explicitly model this possibility.

INFORMATION-GAP ROBUSTNESS ANALYSIS

Information-gap or info-gap theory [2] provides a quantified theory of robustness. A *robust* option is one that performs well even under future conditions that deviate from our best estimate. Robustness analysis is thus a tool for handling uncertainty as it influences engineering decisions. Info-gap also examines *opportunity*, the propitious side to uncertainty, by helping to identify options that may be prone to desirable outcomes under uncertain conditions.

The vast majority of previous treatments of uncertainty in flood risk management decisions have relied upon probabilistic uncertainty representation and normative decision making based upon maximisation of expected utility. The main criticisms of this approach (see for example refs [3] and [6]) relate to the estimation of probability distributions under conditions of severe uncertainty [12] and the impossibility of deriving rules for rational choice in group situations [1], which flood risk management decisions almost always are. In practice populations of data are finite and there are practical limits to the quantity of information that can be elicited from experts. In both cases some further assumption is required in order to construct a probability distribution. The problem becomes particularly acute in situations of very scarce data and where experts express indecision about an uncertain quantity and cannot be induced to provide further information in the appropriate probabilistic format. A recourse of the probabilistic analyst in the face of indeterminacy is to adopt a uniform probability distribution. However, as Keynes [8] demonstrated, incautious adoption of the uniform distribution can lead to contradictions.

In response to these criticisms a number of alternatives have been proposed [11], which in particular seek to exploit the potential of set-based uncertainty representations to deal represent ambiguous information. The most noteworthy of these approaches in terms of their application in hydraulic engineering and hydrology is fuzzy set theory, but as Klir and Smith [10] demonstrate, fuzzy set theory and probability theory can be generalised within the theory of imprecise probabilities [13].

All of these approaches rely upon some normalised measure being applied over the space of possibilities. Info-gap theory, but contrast does not employ normalised measure at all, so does not fit anywhere within the information-theoretic hierarchy of theories of uncertainty developed by Klir [9]. Rather, uncertainty is represented by a family of nested sets bounding the variation of system behaviour about some nominal value $\tilde{u}$. The size of the possible departure from $\tilde{u}$ (in other words the horizon of uncertainty) is parameterized by a hyper-parameter, α. Each value contained within a set $U(\alpha, \tilde{u})$ represents a possible outcome. In an estuarine flood risk management problem, u might be the rate of increase in relative mean sea level over the appraisal period and $\tilde{u}$ our current best estimate of this rate. Info-gap theory examines the performance of a set of alternative acts $\{d\}$ (which might include the acts of raising defences at a certain time by a certain amount, of implementing flood storage, of constructing a barrier) as u varies within the set $U(\alpha, \tilde{u})$, which gets progressively bigger as α increases. No attempt is made to apply any form of measure function, such as a probability density function, over α. Instead the approach examines the performance of design options at different horizons of uncertainty. An assumption remains that values of u become increasingly unlikely as they diverge from $\tilde{u}$.

For each act d and value of u there will be a corresponding reward $R(d, u)$, which is a scalar measure of how the act performs under the conditions defined by u. The reward function might, for example, be the Net Present Value that is yielded by an act under a given set of conditions. At a given horizon of uncertainty α there will be a set of possible performances (which collapses to a singular value $R(d, \tilde{u})$ at $\alpha = 0$), and within that set there will be minimum and maximum levels of performance.

The robustness and opportunity functions represent how these lower and upper bounds on possible performance vary with α. They are defined in terms of two performance targets r_c and r_w. Info-gap is a theory of robust satisficing, in that it seeks to identify acts that perform acceptably well under a wide range of conditions, in contrast to normative decision theory, which seeks to maximise expected utility under assumed conditions. The level of performance that is deemed acceptable is denoted r_c. The robustness function, $\hat{\alpha}(d, r_c)$, measures of the maximum uncertainty that can be borne whilst ensuring a critical level of reward, r_c:

$$\hat{\alpha}(d, r_c) = \max\left\{\alpha : \min_{u \in U(\alpha, \tilde{u})} R(d, u) \geq r_c\right\} \tag{2.1}$$

Only at $\alpha = 0$ can the nominal level of performance $R(d, \tilde{u})$ be guaranteed. For any value of $\alpha > 0$ it is clear that $(min_{u \in U(\alpha, \tilde{u})} R(d, u)) < R(d, \tilde{u})$. There is a trade-off here, in that robustness is sacrificed as the requirement for reward becomes increasingly demanding.

The robustness function reflects system performance with respect to the pernicious effects of uncertainty. However, uncertainty can also yield unexpectedly good reward, which is represented with the opportunity function, $\hat{\beta}(d, r_w)$, a measure of the minimum level of uncertainty required to enable a 'windfall' level of reward, r_w:

$$\hat{\beta}(d, r_w) = \min\left\{\alpha : \max_{u \in U(\alpha, \tilde{u})} R(d, u) \geq r_w\right\} \tag{2.2}$$

Since the robustness function expresses immunity against failure 'bigger is better'. Conversely, when considering the opportunity function 'big is bad' [2]. Typically values for windfall reward will be far greater than those for critical reward. In addition the different behaviours of the robustness and opportunity functions for alternative decision options provides an insight into the potential positive and negative impacts of uncertainty.

Info-gap analysis has found applications in disciplines as diverse as economics, anti-terrorism and environmental protection [2], all of which are subject to severe uncertainties. Since flood risk management decisions can also be subject to severe uncertainties, making the application of info-gap theory in this domain particularly appealing.

EXAMPLE

Decision problem

Consider an estuary, with property located in the adjacent floodplain. Relative mean sea level is increasing, as is the vulnerability of the properties (in terms of the damage caused by a given depth of flood water). Flood risk managers must choose which of set of flood risk management options $d_i : i \in 1, \ldots, N$ to implement in response to these changes.

For a given option, conventional economic appraisal compares the beneficial reduction in expected damage relative to some base case d_0 with the cost of implementation. A standard performance (or "reward") measure is net present value (NPV). Let $r_{i,y}$ and $c_{i,y}$ denote respectively the expected annual damage and costs of implementation associated with option i in year $y \in [0, T)$, where T is the duration of the appraisal period in years. Assuming perfect knowledge of future conditions, NPV would be a function only of the option d_i. In order to take uncertainty regarding those conditions into account, we parameterise also on u, a vector specifying those conditions. The Net Present Value of option d_i is then given by equation (6.1), where $r_{i,y}$ and $c_{i,y}$ must also be defined in terms of u..

$$R(d_i, u) = \sum_{y=0}^{N-1} \frac{1}{(1+s)^y} \left[(r_{0,y} - r_{i,y}) - c_{i,y} \right] \tag{6.1}$$

Info-gap uncertainty model

We will consider three sources of uncertainty: cost error u_c, rate of increase of vulnerability u_v and rate of increase in mean sea level u_z, $u = [u_c, u_v, u_z]^T$.

Costing is notoriously difficult, and all the more so in a strategic planning context where it is only possible to develop indicative costs. We express costing error in terms of cost overrun. Values of u_c of 0, -50% and 100% represent, respectively, a perfect cost estimate and estimates of twice and half actual costs. HM Treasury guidelines suggest that projects should show positive net benefit under a 60% cost overrun. This value is taken as a best estimate value for cost error. A recent summary of cost overruns on UK highway construction projects nearing completion included values in the interval [-14%, +220%]. These are taken as indicative for construction in general. Since the costing used in strategic analysis is necessarily less detailed it might be expected that the errors could be larger. We extend the range of possible error to account for this, setting the bounds at $\alpha = 2$ to [-50%,400%] with bounds for other values of α found by linear interpolation. The result is an asymmetric model in which the upper bound on cost error departs more rapidly from the best estimate with increasing α than the lower bound.

Expected damage is a function not only of hydrological and hydraulic terms, but also of the value of assets at risk in the floodplain, which we refer to as "vulnerability". We expect this to increase as a result of economic growth, but are uncertain as to the rate of increase. The annual compound rate of increase in vulnerability u_v is assumed to be constant over the appraisal period. The influence of economic growth on flood risk was explored in by Evans et al. [4], from which we obtain likely multipliers on flood risk for the 2080s ranging from 6.6 to 36. These values, converted to annual rates of growth assuming compound growth, were assigned $\alpha = 1$. The best estimate was taken as the mean of these values, $u_v = 3.48\%$.

In coastal and estuarial waters, a major driver of changing flood risk is increasing relative mean sea level. The difficulty of forecasting this rate over strategic planning time scales is a major source of uncertainty. Relative mean sea level is assumed to increase linearly over the appraisal period with annual rate u_zm/year. IPCC [7] suggests a best estimate rate of regional time-mean sea level rise of 6.4*mm/year*, and the minimum and maximum rates suggested are 2.1*mm/year and* 10.1*mm/year* respectively. These were taken as the bounds at an uncertainty level corresponding with $\alpha = 1$. In addition to these values, at $\alpha = 2$ the present day rate of sea level rise, 1.6*mm/year*, was taken as a lower bound and combined with a plausible upper extreme of 22*mm/year*. Exponential curves

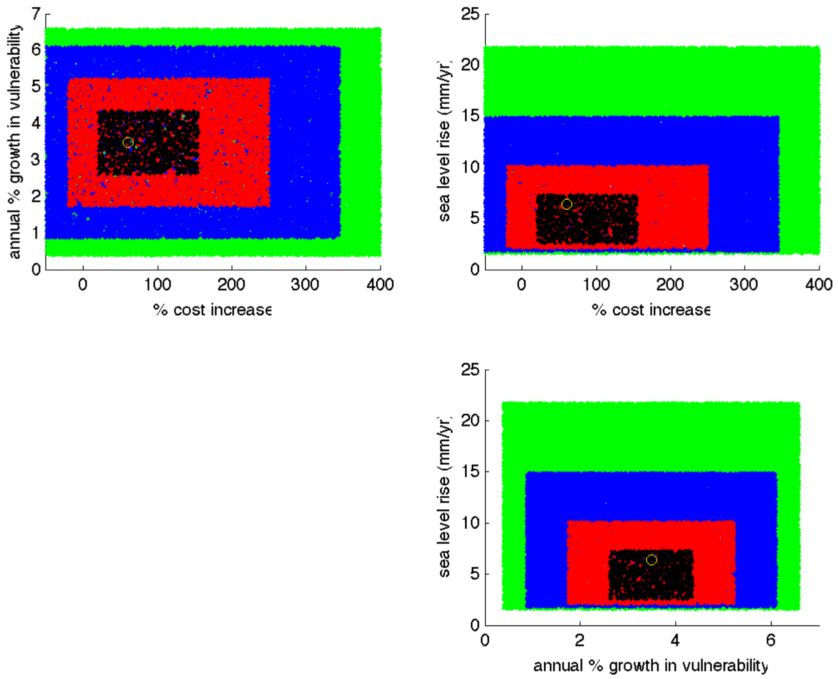

Figure 1 Info-gap uncertainty model. Boxes show the boundaries of α=0.1, 0.25, 0.5, 1.0, 1.5, 2.0. Also plotted are trajectories of minimum and maximum performance (corresponding with robustness and opportuneness) through uncertain parameter space. Traces start from the best estimate ($\alpha = 0$) and diverge as alpha increases. Solid lines show the parameters associated with the minimum performance at a given alpha (robustness), while dashed lines show the maximum (opportuneness).

were fitted to these data, giving an uncertainty model which scales non-linearly and asymmetrically from the best estimate with increasing alpha. The behaviour of the resulting info-gap uncertainty model is depicted graphically in Figure 1, where coloured boxes indicate the nested sets associated with particular values of α.

Options

Four flood defence options are considered. These are:

1. Raise defences in 2025 to levels found to be optimal assuming an annual rate of increase of relative mean sea level at the mouth of the Thames consistent with current best estimates of 2100 level. (Assume that sea level will rise at a constant rate between now and 2100.)
2. Raise defences in 2025 to levels found to be optimal assuming an annual rate of increase of relative mean sea level at the mouth of the Thames consistent with the upper bound on the IPCC range of 2100 levels.
3. Raise defences in 2050 to levels optimal for actual sea level rise, which is assumed to be known by the time works must begin.
4. As 3, but implement temporary works in 2025 to increase the standard of defence. It is assumed that, since the design life of these defences is shorter than those of

permanent works, the cost will be considerably lower. The reliability of temporary and permanent works is assumed to be the same.

Options 3 and 4 are intended to capture the assumption that scientific knowledge regarding sea level rise will improve with time. We model the situation of reduction in uncertainty through scientific research, somewhat simplistically, by assuming that the rate of increase in relative mean sea level will be known perfectly by the 2040s, allowing defences to be built in 2050 which are optimised to the exact actual rate of increase.

Risk and cost models

The risk model used in this example analysis is based on the UK Environment Agency IA8 model as described by Gouldby et al. [5]. A very simple cost model is used, with a fixed component representing mobilisation cost and a two-part piecewise linear variable component such that crest raising becomes more expensive above a threshold. We assume that the cost of implementing crest level raising by temporary works is 30% of permanent works of the same scale, and that their removal costs one third of much as their construction cost. It would be straightforward to update this analysis to use an improved cost model, which could be derived from a combination of representative bills of quantities and data on past works of similar type.

RESULTS

A traditional economic appraisal might rank options on the basis of their Net Present Value. Under the assumption that the rate of sea level rise, rate of growth in vulnerability and the error in cost estimates all match our current best estimates (that is, that $u = \tilde{u}$), Option 4 (temporary work in 2025, permanent works in 2050) gains first place by a small margin: it provides much of the protection of option 1 but at lower present value cost as the cost of permanent defence raising is deferred by 25 years. Option 3 (do nothing until 2050) performs least well, as it provides no protection from increasing expected annual damage due to rising sea levels through the period between 2025 and 2050. Option 2, a precautionary crest level raise in 2025, loses out marginally to option 1 as would be expected since option 1 was optimised for these conditions. The option ranking is therefore: $d_4 > d_1 > d_2 > d_3$.

Error! Reference source not found. shows robustness curves for the four options. These curves show how minimum performance deteriorates as the horizon of uncertainty expands. At $\alpha = 0$ the set of possibilities $U(\alpha, \tilde{u})$ collapses to contain only best estimate conditions, $U(\alpha, \tilde{u}) = \{\tilde{u}\}$. As α increases and $U(\alpha, \tilde{u})$ encompasses an increasingly wide range of possible conditions, the guaranteed minimum performance drops rapidly for all options. Varying robustness curve gradient between options leads to curve crossing, implying a change in preference ordering for a decision based on minimum performance at a given horizon of uncertainty. Notice in particular that option 3 (permanent defence raising in

2050), which initially performs worst, deteriorates less quickly with increasing α and at high alpha performs almost as well as option 4 (early temporary and late permanent works).

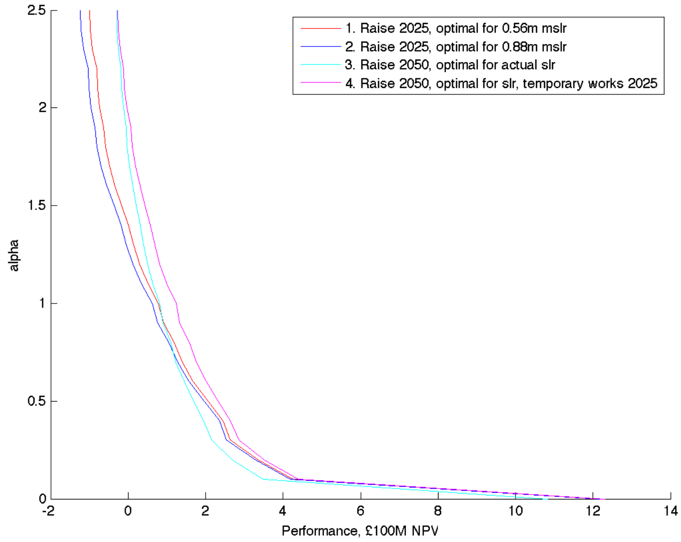

Figure 2 Robustness curves at a discount rate of 4%. The lines show the minimum performance $R(d,u)$ that could occur at a given horizon of uncertainty $u \in U(\alpha,\tilde{u})$.

To fully understanding the form of the robustness curves requires some exploration of the data from which they are derived. Recall that these curves show, respectively, the minimum and maximum performance associated with an option at a given horizon of uncertainty α. These minima and maxima relate to particular points in the three-dimensional uncertain parameter space $U(\alpha,\tilde{u})$. Ordered by increasing α, the set of points associated with a robustness curve describes a trajectory through parameter space. Visualisations of these trajectories can help reveal which of the uncertainties being considered are controlling the form of the robustness curve.

Figure 3 shows such a visualisation for our example. Each subfigure shows a projection of the trajectories onto a pair of axes. All four options are plotted here. That their trajectories are in general not distinct is the first useful piece of information: all of the options respond to the uncertainties considered in the same way. In the present example this is unsurprising, as the options are structurally similar.

The boxes shown in Figure 1 denote the surface of the uncertainty model, $U(\alpha,\tilde{u})$, for a few values of α. We see in subfigure (b) that robustness tracks the corner of the uncertainty model associated with *minimum* rate of sea level rise and *minimum* rate of growth in vulnerability, from which we deduce that these uncertainties are controlling the form of the robustness curve. These uncertainties also exert control over the form of the opportuneness curve, with here the *maximum* rates of sea level rise and vulnerability growth being associated with the maximum opportunity for windfall benefit. From Figures 3(a) and (c) it is clear that cost error is also significant in terms of robustness, but not at all in terms of opportuneness as indicated by the random fluctuations. Close examination near the best estimate indicates that high cost overrun becomes a significant influence on robustness as α increases.

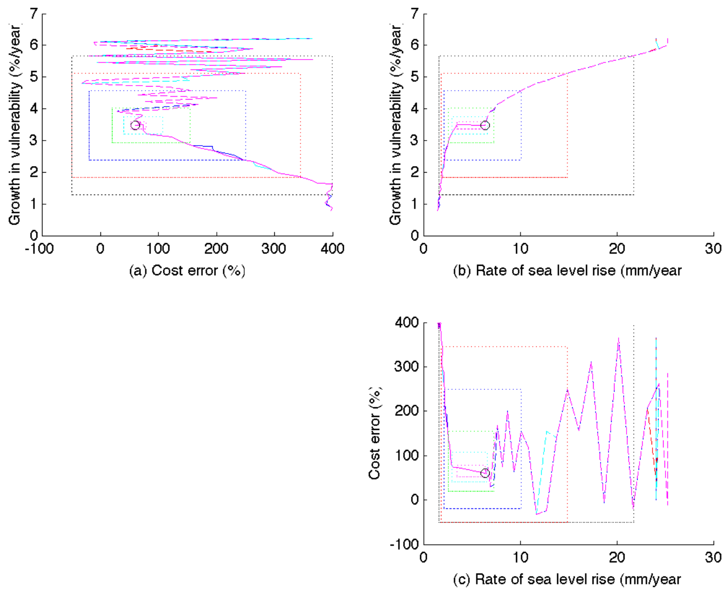

Figure 3 Info-gap uncertainty model. Boxes show the boundaries of α=0.1, 0.25, 0.5, 1.0, 1.5, 2.0. Also plotted are trajectories of minimum and maximum performance (corresponding with robustness and opportuneness) through uncertain parameter space. Traces start from the best estimate ($\alpha = 0$) and diverge as alpha increases. Solid lines show the parameters associated with the minimum performance at a given alpha (robustness), while dashed lines show the maximum (opportuneness).

We find that the futures in which our options perform worst are associated with low rates of sea level rise and increase in vulnerability. Conversely, high rates of increase appear as an opportunity to win big. These results may at first seem counterintuitive, as we are used to thinking of sea level rise and increasing vulnerability as a source of “risk”. In this analysis, however, we are exploring not expected damage, but the net present value of engineering interventions.

Risk and opportunity are two sides of the same coin. All four options perform very well under best estimate conditions, with large positive NPV, indicating that defence raising is, in a densely populated area such as the study area, a cost effective means of reducing flood risk. The present value risk reduction is far greater than the cost of constructing the defences required to obtain it. Deviations from best estimate parameter values that reduce benefit reduce performance more rapidly than those that increase cost. As mean sea level and vulnerability decrease, defence raising becomes increasingly expensive relative to the savings possible in present value damage. The converse is also true: high rates of sea level rise and growth in vulnerability increase the benefit to be gained by the relatively cheap mechanism of dyke building.

At high values of α, the options in which permanent works are implemented later offer better worst-case performance. The cases from which these results arise involve low potential benefit and high cost overrun. The result is that the opportunity cost associated with constructing defences is in these cases very high and delaying work is advantageous. At lower horizons of uncertainty losses from rising mean sea level prior to 2050 are sufficient to outweigh this.

It is noteworthy that option 4, involving delayed permanent defence raising but early temporary works, was not one of the initial set of options. It was added as a result of the examination of the behaviour of the other options. This is significant because it indicates that info-gap analysis provides a tool not only useful in analysing options once they have been defined, but also in supporting the process of designing options.

CONCLUSIONS

We have described the formulation of a flood risk management decision relating to the timing of interventions. By modelling our assumptions about the way in which uncertainties will change in time, we have been able to explore the possible impact of these changes on the performance of different options (using Net Present Value as the performance measure).

We have applied info-gap analysis to these options. The specific results of this analysis were that under conditions of high uncertainty about rates of change of key variables, options involving delaying in order to utilise improving scientific knowledge can provide better guaranteed minimum performance than those which involve implementing measures based on current best estimates of future conditions.

Examination of the behaviour of the options initially tested generated insights which suggested that a hybrid option might offer some of the advantages of both schemes. This option (option 4) did indeed provide better guaranteed minimum performance over the range of horizons of uncertainty considered.

This application confirmed that info-gap analysis generates information of potential value in choosing between options when uncertainty regarding the future values of key variables is high and the ranking of options is not possible. It also suggested that info-gap analysis may be a valuable tool in the process of designing options for robustness.

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